Advanced Mathematical Models & Applications
Vol.4, No.2, 2019, pp.167-178

MATHEMATICAL MODELING OF CHLORINE DECAY THROUGH WATER AND INTERMEDIATE PSEUDOMONAS AERUGINOSA IN MULTIPLE-COMPARTMENT ISOTHERMAL REACTOR

O. Adedire*, **J.N. Ndam**

Department of Mathematics, University of Jos, Nigeria

Abstract. This research work investigates chlorine concentration decay profile through multiple-compartment isothermal reactor system using a constructed mathematical model which comprises of both systems of Partial Differential Equations (PDEs) and Ordinary Differential Equations (ODEs). The multiple-compartment isothermal reactor system consists of both bulk fluid of water and intermediate Pseudomonas aeruginosa with humic acid attached to the sieve between each compartment. The PDEs were discretized in space using Method of Lines (MOL) procedure and were solved simultaneously due to coupling at the inter-connected boundaries and due to coupling of reaction terms in some sections of the system. The discretized PDEs together with the associated ODEs results in a total of 134 ODEs which were solved using Matlab ODE solver, ODE 15s due to its efficient way of handling such problems using variable step size. At initial times before attaining steady state, this model predicts the spread and consumption rate of chlorine across all sections in space but the rate of spread and consumption of chlorine gradually reduces as time progressively increases.

Keywords: Mathematical model, chlorine, water, Pseudomonas aeruginosa, multiple-compartment, isothermal system, chemical reactor.

AMS Subject Classification: 65620, 97M10, 93A30, 81T80.

***Corresponding author:** O. Adedire, Department of Mathematics, University of Jos, Nigeria,
e-mail: *dharenss@gmail.com*

Received: 14 June 2019; Revised: 22 June 2019; Accepted: 30 June 2019; Published: 14 August 2019.

1 Introduction

Some reservoirs are constructed for the purpose of water storage but the quality of water can be affected by age of the water thereby creating avenue for contamination if adequate disinfectant is not added. Biofilm starts to form within and around the internal walls of the tank; they are thin layer of cells of organisms such as Pseudomonas aeruginosa. There may be possibility of recontamination even for households that have access to piped water (Levy et al., 2008).

Selleck et al. (1978) found that concentration of a disinfectant is one of the essential changing features behind the effectiveness of disinfecting microorganisms. Chlorine in water is necessary as disinfectant and to act as a residual maintenance because of its strong oxidation power against contaminants such as biofilm of Pseudomonas aeruginosa. Chemical substance as chlorine may be added to water in gaseous, liquid or solid form and it may form disinfection by-products as further contaminants whenever high concentration of organic matter is present. Chlorine is able to destroy some plastic and coating materials increasingly by chemical action and it is corrosive to many metals in the presence of water. Interaction of chlorine with some species of microorganisms often leads to reduction in their concentration (Kie ne et al., 1993). This reduction can be preferably modelled mainly as first and second order decay in the bulk fluid and at the walls (Sharp et al., 1991; Powell et al., 2000).

In literature, some models have been developed to check propagation of disinfectants in drinking water. While some of the models are centred on unsteady state analysis, others are centred on steady state analysis with specific consideration of decay in pipes of water distribution system (Biswas et al., 1993; Vasconcelos et al., 1996). Background to this work came from the work of previous researchers (Chen & Stewart, 1996; Xu et al., 2018; Liou & Kroon, 1987; Mau et al., 1996; Wang & Falconer, 1998; Lee et al., 2018; Mao et al., 2018; Armstrong et al., 2016; Pabst et al., 2016) who have modelled disinfectant decay through different systems.

Chen and Stewart (1996) conducted research to investigate chlorine penetration into artificial biofilms of Pseudomonas aeruginosa and concluded that it was consistent with unsteady reaction diffusion model. They however did not extend their work to multiple-compartment systems which are linked and which are often seen in some reactors with sieve partitions.

Motivation to proceed on this study using P.aeruginosa comes from the fact it is a bacterium commonly present in water sources. While many homes in developing nations often practise water collection from different sources for storage in tanks due to their inability to access continuous flow purified water, some water treatment plants have storage units with sieve partitions which may possibly have attachment of P.aeruginosa due to certain factors. Further motivation for this study is the possibility of lowering one of the global problems of water-borne sickness using a model that can predict transport of biocide through both bulk water and penetration of biofilm of micro-organisms especially in household where disinfectant residual chemical is needed to hinder process of recontamination.

From the foregoing, satisfactory research has not been conducted on the transport of chlorine decay as disinfectant through both bulk water and intermediate Pseudomonas aeruginosa in water storage unit with multiple-compartment chambers. The need to investigate chlorine decay through such multiple-compartment systems arise because it will help to account for the depletion of chlorine in the intermediate zones consisting of P.aeruginosa with humic acid which are not usually considered in one-compartment systems.

In this study, we investigate the decay of chlorine through partitioned multiple-compartment reactor. This is to mathematically model transport of chlorine through the media of both bulk water and Pseudomonas aeruginosa covering the sieve partition of the reactor system simultaneously. The transport of chlorine follows the work of Chen and Stewart (1996) and inactivation kinetics of the Pseudomonas aeruginosa is based on Chick-Watson model (Chick, 1908; Watson, 1908)

$$\frac{d\varphi}{dt} = -k_i \Psi \varphi, \tag{1}$$

where φ is taken as the density of Pseudomonas aeruginosa, Ψ is the concentration of the dissolved chlorine at time t and k_i is the inactivation constant.

The subsequent part of this paper is organized as follows: section 2 deals with the derivation of the model for chlorine transport through both bulk water and biofilm of Pseudomonas aeruginosa. Section 3 deals with numerical simulation of the model. While in section 4, the results and discussion of the model are presented. Conclusion comes up in section 5.

2 Model Development

Hydrolysis of chlorine when put into water produces hydrochloric acid (HCl) and hypochlorous acid (HOCl). Formation of hypochlorite ion (OCl^-) and hydrogen ion (H^+) then occur when HOCl dissociates partially further in water. The disinfection of drinking water occurs through the actions of oxidizing agents HOCl and OCl^- , thus the model equations are obtained in terms of dissolved chlorine concentration HOCl + OCl^- (Biswas et al., 1993).

Figure 1 shows the continuous transport of dissolved chlorine through the compartments of the chemical reactor with intermediate biofilm of Pseudomonas aeruginosa with humic acid attached to two sieve partitions. These sieve partitions allow passage of molecular flow of disinfectant components.

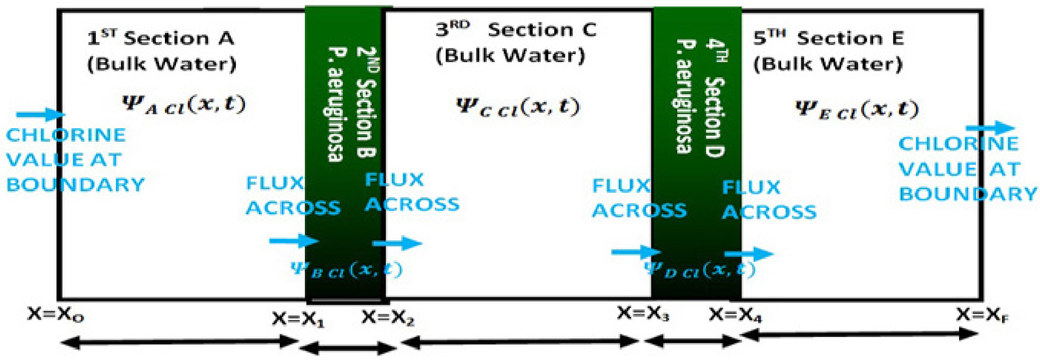

Figure 1: Schematics of bulk water and biofilm of P.aeruginosa in a multiple-compartment system for chlorine transport.

The mass balance over the control volume in each section with assumption of diffusion along x-axis results in the following systems of partial differential equations (PDEs) and ordinary differential equations (ODEs) for both chlorine disinfectant and biofilm of Pseudomonas aeruginosa with humic acid contaminants:

1st section (bulk water):

$$\frac{\partial\Psi_{Acl}(x,t)}{\partial t} = \frac{\partial}{\partial x}[D_A\frac{[\partial\Psi_{Acl}(x,t)]}{\partial x}] - k_{Acl}\Psi_{Acl}(x,t), \qquad x_0 \leq x \leq x_1. \tag{2}$$

Equation (2) has the initial condition

$$\Psi_{Acl}(x,0) = \Psi_{Acl0}(x) \tag{3}$$

and boundary conditions specified as follows:

$$\Psi_{Acl}(x = x_0, t) = \Psi_{Acl0}(t), \tag{4}$$

$$\Psi_{Acl}(x = x_1, t) = \Psi_{Bcl}(x = x_1, t). \tag{5}$$

2nd section (Pseudomonas aeruginosa):

$$\frac{\partial\Psi_{Bcl}(x,t)}{\partial t} = \frac{\partial}{\partial x}[D_B\frac{[\partial\Psi_{Bcl}(x,t)]}{\partial x}] - k_{B1}\varphi_{psB1}\Psi_{Bcl}(x,t) - k_{B2}\varphi_{hmB2}\Psi_{Bcl}(x,t),$$
$$x_1 \leq x \leq x_2. \tag{6}$$

Equation (6) has the initial condition

$$\Psi_{Bcl}(x,0) = \Psi_{Bcl0}(x) \tag{7}$$

and boundary conditions specified as follows:

$$\frac{\partial\Psi_{Bcl}(x=x_1,t)}{\partial x} = \frac{D_{Acl}}{D_{Bcl}}\frac{\partial\Psi_{Acl}(x=x_1,t)}{\partial x}, \tag{8}$$

$$\Psi_{Bcl}(x = x_2, t) = \Psi_{Ccl}(x = x_2, t). \tag{9}$$

3rd section (bulk water):

$$\frac{\partial\Psi_{Ccl}(x,t)}{\partial t} = \frac{\partial}{\partial x}[D_C\frac{[\partial\Psi_{Ccl}(x,t)]}{\partial x}] - k_{Ccl}\Psi_{Ccl}(x,t), \qquad x_2 \le x \le x_3. \tag{10}$$

Equation (10) has the initial condition

$$\Psi_{Ccl}(x,0) = \Psi_{Ccl0}(x) \tag{11}$$

and boundary conditions specified as follows:

$$\frac{\partial\Psi_{Ccl}(x=x_2,t)}{\partial x} = \frac{D_{Bcl}}{D_{Ccl}}\frac{\partial\Psi_{Bcl}(x=x_2,t)}{\partial x}, \tag{12}$$

$$\Psi_{Ccl}(x=x_3,t) = \Psi_{Dcl}(x=x_3,t). \tag{13}$$

4th section (Pseudomonas aeruginosa):

$$\frac{\partial\Psi_{Dcl}(x,t)}{\partial t} = \frac{\partial}{\partial x}[D_D\frac{[\partial\Psi_{Dcl}(x,t)]}{\partial x}] - k_{D1}\varphi_{psD1}\Psi_{Dcl}(x,t) - k_{D2}\varphi_{hmD2}\Psi_{Dcl}(x,t), \quad x_3 \le x \le x_4. \tag{14}$$

Equation (14) has the initial condition

$$\Psi_{Dcl}(x,0) = \Psi_{Dcl0}(x) \tag{15}$$

and boundary conditions specified as follows:

$$\frac{\partial\Psi_{Dcl}(x=x_3,t)}{\partial x} = \frac{D_{Ccl}}{D_{Dcl}}\frac{\partial\Psi_{Ccl}(x=x_3,t)}{\partial x}, \tag{16}$$

$$\Psi_{Dcl}(x=x_4,t) = \Psi_{Ecl}(x=x_4,t). \tag{17}$$

5th section (bulk water):

$$\frac{\partial\Psi_{Ecl}(x,t)}{\partial t} = \frac{\partial}{\partial x}[D_E\frac{[\partial\Psi_{Ecl}(x,t).]}{\partial x}] - k_{Ecl}\Psi_{Ecl}(x,t), \qquad x_4 \le x \le x_5 \tag{18}$$

Equation (18) has the initial condition

$$\Psi_{Ecl}(x,0) = \Psi_{Ecl0}(x) \tag{19}$$

and boundary conditions specified as follows:

$$\frac{\partial\Psi_{Ecl}(x=x_4,t)}{\partial x} = \frac{D_{Dcl}}{D_{Ecl}}\frac{\partial\Psi_{Dcl}(x=x_4,t)}{\partial x}, \tag{20}$$

$$\Psi_{Ecl}(x=x_5,t) = \Psi_{Ecl0}(t). \tag{21}$$

Observe that the boundary conditions given in (5), (9), (13) and (17) are connected boundary conditions linking activities of chlorine in one section with the preceding section in a way showing relationship at the boundary.

Mass balances with respect to biofilm of Pseudomonas aeruginosa and humic acid when the growth and cell detachment are neglected give rise to the following ODEs in the second section

$$\frac{d\varphi_{psB1}(t)}{dt} = -k_{B1}\varphi_{psB1}(t)\Psi_{Bcl}(x_0,t), \qquad t \ge 0 \quad x_0 \in [x_1,x_2], \tag{22}$$

$$\frac{d\varphi_{hmB2}(t)}{dt} = -k_{B2}\varphi_{hmB2}(t)\Psi_{Bcl}(x_0,t), \qquad t \ge 0 \quad x_0 \in [x_1,x_2] \tag{23}$$

with initial conditions

$$\varphi_{psB1}(t_0) = \varphi_{psB1O}, \qquad x_1 \leq x \leq x_2, \tag{24}$$

$$\varphi_{hmB2}(t_0) = \varphi_{psB2O}, \qquad x_1 \leq x \leq x_2. \tag{25}$$

In the fourth section, the mass balances with respect to Pseudomonas aeruginosa and humic acid when the growth and cell detachment are neglected give rise to the following ODEs

$$\frac{d\varphi_{psD1}(t)}{dt} = -k_{D1}\varphi_{psD1}(t)\Psi_{Dcl}(x_0,t), \qquad t \geq 0 \quad x_0 \in [x_1,x_2], \tag{26}$$

$$\frac{d\varphi_{hmD2}(t)}{dt} = -k_{D2}\varphi_{hmD2}(t)\Psi_{Dcl}(x_0,t), \qquad t \geq 0 \quad x_0 \in [x_1,x_2] \tag{27}$$

with initial conditions

$$\varphi_{psD1}(t_0) = \varphi_{psD1O}, \qquad x_3 \leq x \leq x_4, \tag{28}$$

$$\varphi_{hmD2}(t_0) = \varphi_{psD2O}, \qquad x_3 \leq x \leq x_4, \tag{29}$$

where the terms of (2), (10), and (18) from left hand side to the right hand side denote the net accumulation, diffusion and first order decay of chlorine in the bulk fluid of water contained in sections A, C and E. The terms of (6) and (14) from left denote net accumulation, diffusion, second order decay of chlorine due to Pseudomonas aeruginosa cell and second order decay of chlorine due to humic acid present in the regions B and D containing Pseudomonas aeruginosa respectively.

Also observe that the boundary conditions given by (8), (12) and (16) are boundary conditions showing continuity of flux of chlorine from one section to another which are represented by Neumann boundary conditions. This shows that the flux of one section is connected to another as chlorine is being tranported through all the compartments.

The terms φ_{psB1} and φ_{psD1} are concentration density of Pseudomonas aeruginosa, φ_{hmB2} and φ_{hmD2} are concentration of humic acid, Ψ_{Bcl} and Ψ_{Dcl} are concentrations of chlorine, k_{B1} and k_{D1} are the rate constant of reaction between chlorine and Pseudomonas aeruginosa, k_{B2} and k_{D2} are the rate constant of reaction between chlorine and humic acid, in the second and fourth sections of the reactor respectively. Equations (2)-(29) consist of systems of five PDEs and four ODEs which are coupled and must be solved simultaneously.

3 Numerical Simulation

The numerical solution of the proposed model for the multiple-compartment system is achieved using the numerical procedure called Method of Lines (MOL). Here the spatial derivatives in the PDEs are replaced with finite differences (FDs) approximations. Using finite central difference scheme, equations (2) when discretized in space becomes

$$\frac{d\Psi_{Acl}(t)_i}{dt} = D_A[\frac{\Psi_{Acl}(t)_{i+1} - 2\Psi_{Acl}(t)_i + \Psi_{Acl}(t)_{i-1}}{\Delta x^2}] - k_{Acl}\Psi_{Acl}(t)_i \tag{30}$$

with initial condition discretized as

$$\Psi_{Acl}(t=0)_i = \Psi_{Acl0} \tag{31}$$

and boundary conditions discretized as follows:

$$\Psi_{Acl}(t)_{i=1} = \Psi_{Acl0}, \tag{32}$$

$$\Psi_{Acl}(t)_{M_{(1)}} = \Psi_{Bcl}(t)_{M_{(1)}}, \;\; 1 \leq i \leq M_{(1)}. \tag{33}$$

Equation (6) is discretized in space as:

$$\frac{d\Psi_{Bcl}(t)_i}{dt} = D_B[\frac{\Psi_{Bcl}(t)_{i+1}-2\Psi_{Bcl}(t)_i+\Psi_{Bcl}(t)_{i-1}}{\Delta x^2}] - k_{B1}\varphi_{psB1}\Psi_{Bcl}(t)_i - k_{B2}\varphi_{hmB2}\Psi_{Bcl}(t)_i \tag{34}$$

and its initial condition discretized as:

$$\Psi_{Bcl}(t=0)_i = \Psi_{Bcl0} \tag{35}$$

and boundary conditions discretized as follows:

$$\frac{\Psi_{Bcl}(t)_{M_{(1)}}-\Psi_{Bcl}(t)_{M_{(1)}-1}}{\Delta x} = \frac{D_{Acl}}{D_{Bcl}}\frac{\Psi_{Acl}(t)_{M_{(1)}}-\Psi_{Acl}(t)_{M_{(1)}-1}}{\Delta x}, \tag{36}$$

$$\Psi_{Bcl}(t)_{M_{(2)}} = \Psi_{Ccl}(t)_{M_{(2)}}, \;\; 1 \leq i \leq M_{(2)}. \tag{37}$$

Discretization of (10) in space results in

$$\frac{d\Psi_{Ccl}(t)_i}{dt} = D_C[\frac{\Psi_{Ccl}(t)_{i+1}-2\Psi_{Ccl}(t)_i+\Psi_{Ccl}(t)_{i-1}}{\Delta x^2}] - k_{Ccl}\Psi_{Ccl}(t)_i \tag{38}$$

with its initial condition discretized as:

$$\Psi_{Ccl}(t=0)_i = \Psi_{Ccl0} \tag{39}$$

and boundary conditions discretized as follows:

$$\frac{\Psi_{Ccl}(t)_{M_{(2)}}-\Psi_{Ccl}(t)_{M_{(2)}-1}}{\Delta x} = \frac{D_{Bcl}}{D_{Ccl}}\frac{\Psi_{Bcl}(t)_{M_{(2)}}-\Psi_{Bcl}(t)_{M_{(2)}-1}}{\Delta x}, \tag{40}$$

$$\Psi_{Ccl}(t)_{M_{(3)}} = \Psi_{Dcl}(t)_{M_{(3)}}, \;\; 1 \leq i \leq M_{(3)}. \tag{41}$$

Equation (14) is discretized as:

$$\frac{d\Psi_{Dcl}(t)_i}{dt} = D_D[\frac{\Psi_{Dcl}(t)_{i+1}-2\Psi_{Dcl}(t)_i+\Psi_{Dcl}(t)_{i-1}}{\Delta x^2}] - k_{D1}\varphi_{psD1}\Psi_{Dcl}(t)_i - k_{D2}\varphi_{hmD2}\Psi_{Dcl}(t)_i \tag{42}$$

with its initial condition discretized as:

$$\Psi_{Dcl}(t=0)_i = \Psi_{Dcl0} \tag{43}$$

and its boundary conditions discretized as:

$$\frac{\Psi_{Dcl}(t)_{M_{(3)}}-\Psi_{Dcl}(t)_{M_{(3)}-1}}{\Delta x} = \frac{D_{Ccl}}{D_{Dcl}}\frac{\Psi_{Ccl}(t)_{M_{(3)}}-\Psi_{Ccl,}(t)_{M_{(3)}-1}}{\Delta x}. \tag{44}$$

$$\Psi_{Dcl}(t)_{M_{(4)}} = \Psi_{Ecl}(t)_{M_{(4)}}, \;\; 1 \leq i \leq M_{(4)}. \tag{45}$$

Equation (18) is discretized as:

$$\frac{d\Psi_{Ecl}(t)_i}{dt} = D_E[\frac{\Psi_{Ecl}(t)_{i+1}-2\Psi_{Ecl}(t)_i+\Psi_{Ecl}(t)_{i-1}}{\Delta x^2}] - k_{Ecl}\Psi_{Ecl}(t)_i \tag{46}$$

with its initial condition discretized as:

$$\Psi_{Ecl}(t=0)_i = \Psi_{Ecl0} \tag{47}$$

and boundary conditions discretized as follows:

$$\frac{\Psi_{Ecl}(t)_{M_{(4)}}-\Psi_{Ecl}(t)_{M_{(4)}-1}}{\Delta x} = \frac{D_{Dcl}}{D_{Ecl}}\frac{\Psi_{Dcl}(t)_{M_{(4)}}-\Psi_{Dcl}(t)_{M_{(4)}-1}}{\Delta x}, \tag{48}$$

$$\Psi_{Ecl}(t)_{M_{(5)}} = \Psi_{Ecl}(t)_{M_{(5)}}, \;\; 1 \leq i \leq M_{(5)}. \tag{49}$$

where $M_{(1)}, M_{(2)}, M_{(3)}, M_{(4)}$ and $M_{(5)}$ represent number of grid points in each section of the system taken to be 26 in each section. Observe that the PDEs (2), (6), (10), (14) and (20) have each becomes semi-discrete systems of ODEs (30), (34), (38), (42) and (46) in one independent variable t. When appropriate auxiliary conditions are substituted into the main semi-discrete systems and further simplifications are made, then the resulting systems of ODEs are solved together with four system of ODEs (22), (23), (26), (27) using Matlab ODE 15s solver due to its efficient way of handling such systems of ODEs with variable step size.

4 Results and Discussion

Motivation for the choice of multiple-compartment reactor sample configuration came from the measurements of one of the samples of multiple-compartment reactors fabricated for household use. The parameter values used for both bulk fluid and biofilm of Pseudomonas aeruginosa cells with humic acids are calculated as well as obtained from literature (Perry, 1973; De Beer et al., 1994; Chen and Stewart 1996). The diffusivity of chlorine in bulk water (D_A, D_C, D_E) and chlorine diffusivity in Pseudomonas aeruginosa cells with humic acid (D_B, D_D) are 2.67×10^{-5} and 2.5×10^{-5} cm^2/s respectively. We assume that the biofilms in each section of B($X_2 - X_1$) and ($X_4 - X_3$) has uniform thickness of $428\mu m$ and the length of each bulk water section is 280 cm. Initial concentrations of P. aeruginosa cells with humic acid in B and D are taken to be 901mg/L and 10,000mg/L respectively. The first order reaction rate constant of chlorine in bulk water ($k_{ACl}, k_{CCl}, k_{ECl}$) is taken to be 7.5×10^{-6} s^{-1} while 1.1×10^{-3} L mg s^{-1} and 3.7×10^{-6} L mg s^{-1} are the second order reaction rate constants of chlorine in $P.$ aeruginosa cells and humic acid respectively. The chlorine concentration Ψ_{Acl0} at the left boundary is taken as $20mg/L$. After further simplifications, the results from simulation of semi-discrete equations (30)-(49) are displayed only for chlorine concentration decay profile for 18 minutes as shown in Figures 2-6. The results are also displayed for 300 minutes as shown in Figures 7-11.

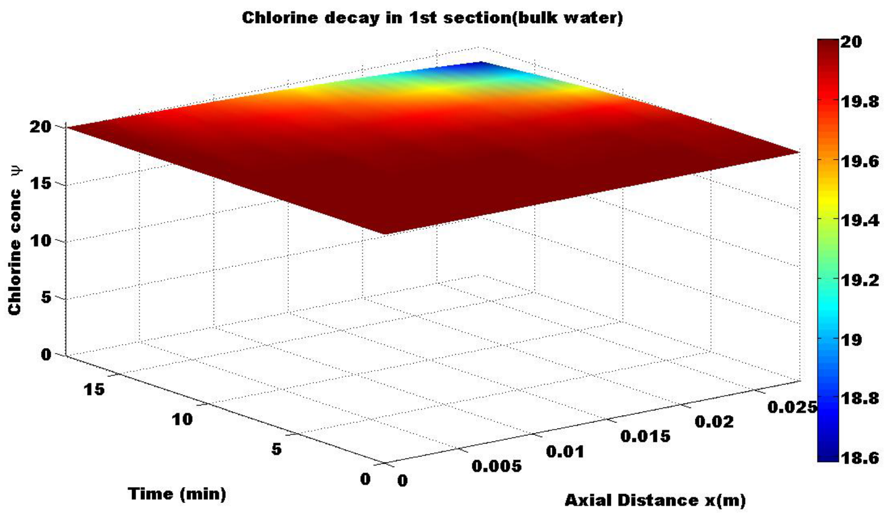

Figure 2: Chlorine concentration profile for 1st section (bulk water) at time $t = 18$ mins

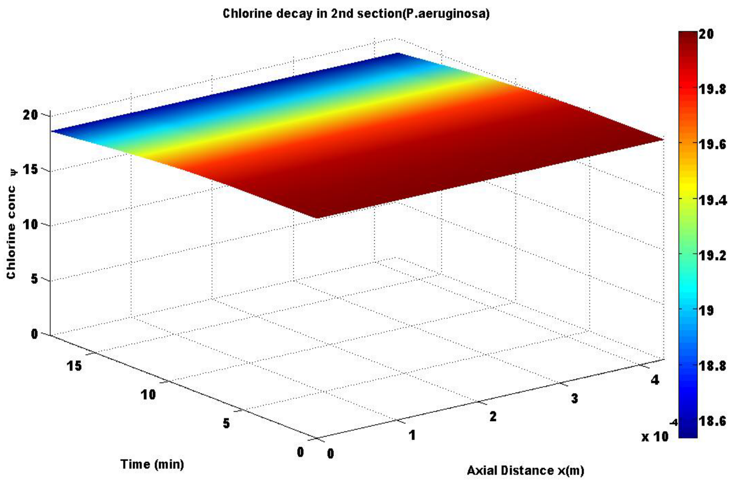

Figure 3: Chlorine concentration profile for 2nd section (P. aeruginosa) at time $t = 18$ mins

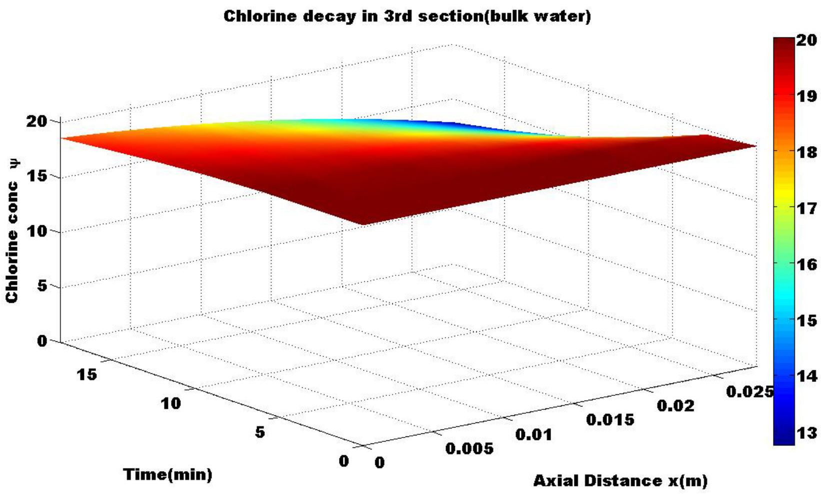

Figure 4: Chlorine concentration profile for 3rd section (bulk water) at time $t = 18$ mins

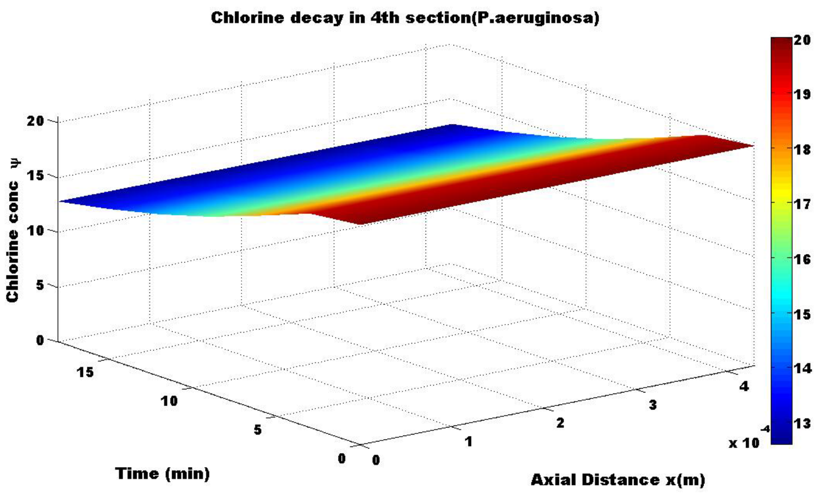

Figure 5: Chlorine concentration profile for 4th section (P. aeruginosa) at time $t = 18$ mins

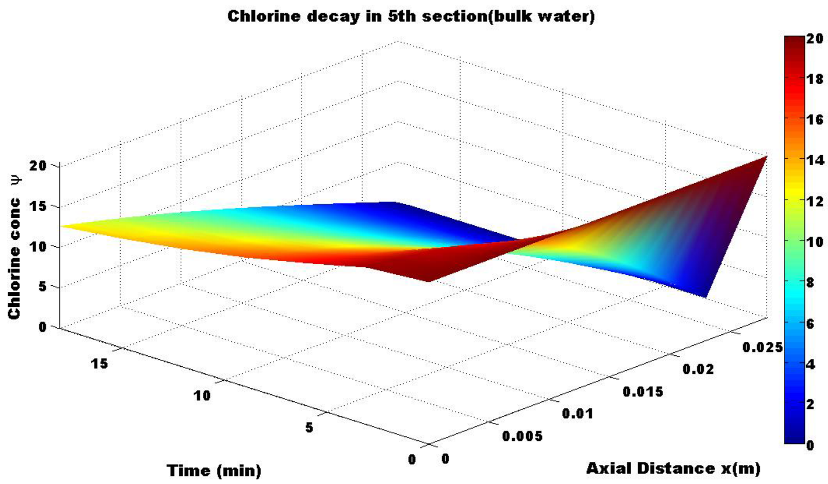

Figure 6: Chlorine concentration profile for 5th section (bulk water) at time $t = 18$ mins

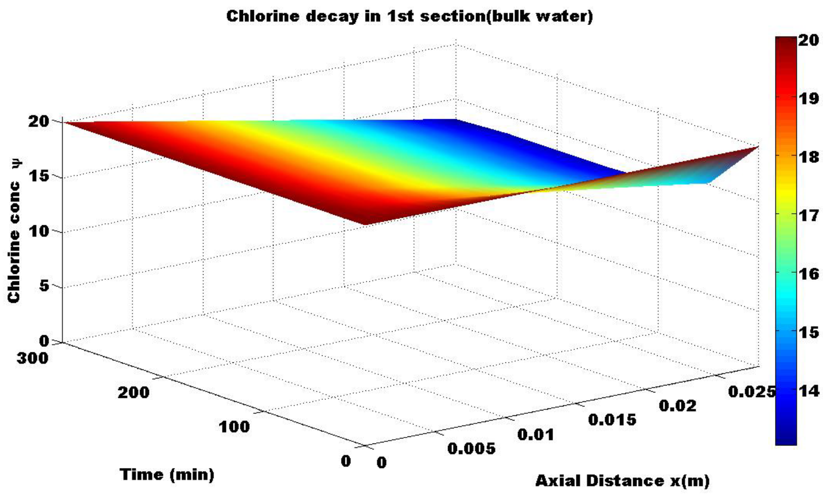

Figure 7: Chlorine concentration profile for 1st section (bulk water) at time $t = 300$ mins

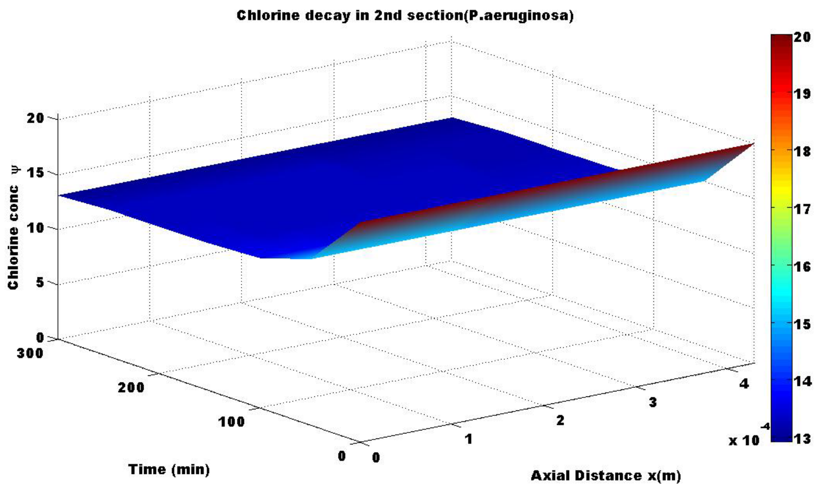

Figure 8: Chlorine concentration profile for 2nd section (P. aeruginosa) at time t = 300 mins

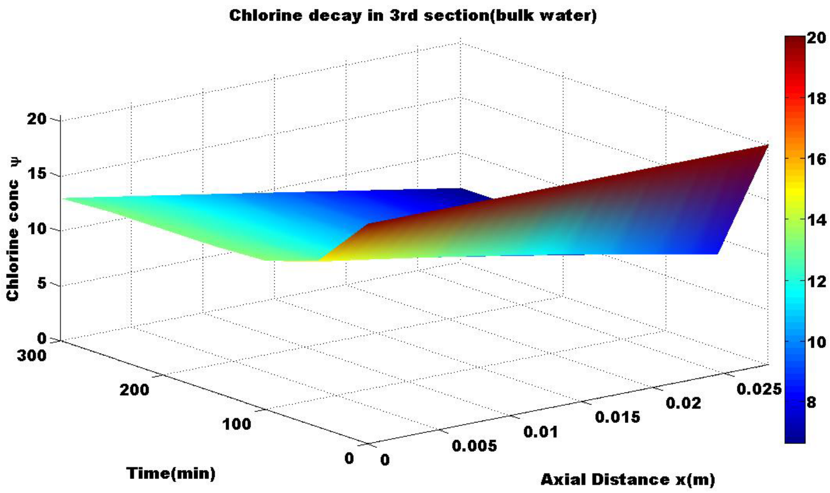

Figure 9: Chlorine concentration profile for 3rd section (bulk water) at time t = 300 mins

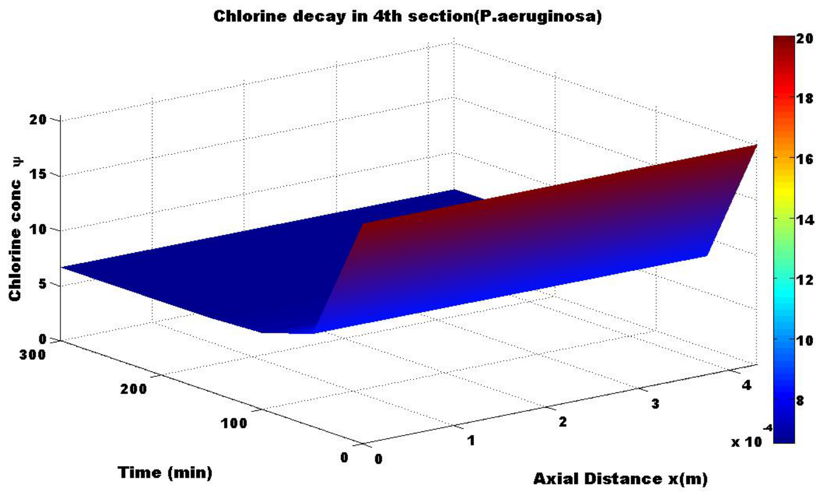

Figure 10: Chlorine concentration profile for 4th section (P. aeruginosa) at time t = 300 mins

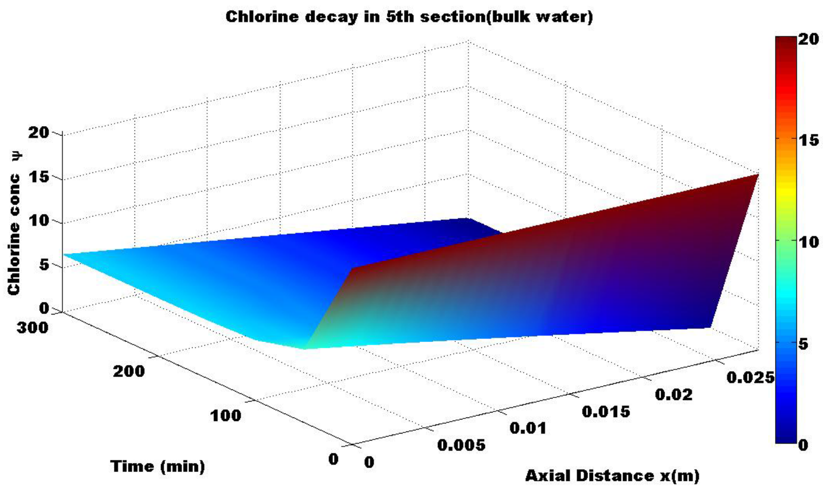

Figure 11: Chlorine concentration profile for 5th section (bulk water) at time t = 300 mins

At time $t = 18$ minutes, the value of chlorine at the interface between the first section (bulk water) and the second section (P. aeruginosa) was 18.58mg/L. While penetrating through the thickness of Pseudomonas aeruginosa, the concentration profile at the boundary between the second section(P. aeruginosa) and the third section(bulk water) was found to be 18.53mg/L at $t = 18$ minutes. At the interface between the third section (bulk water) and the fourth section (P. aeruginosa), the value obtained for chlorine concentration was 12.75mg/L. A value of 12.60mg/L of chlorine was obtained at the point of interaction between the fourth section consisting of P. aeruginosa and the fifth section consisting of bulk water.

Similar numerical simulation was performed for the same set of equations and parameters as aforementioned at $t = 300$ minutes, and the results are shown in Figures 7-11. The concentration of chlorine values of 13.34mg/L for the interface between first and second sections, 13.23mg/L for interface between second and third sections, 6.66mg/L for interface between third and fourth sections and 6.55mg/L for the boundary between fourth and fifth sections were obtained.

Using the model, calculated average quantity of chlorine at mid-point of the fifth section of the multiple-compartment system was 3.27mg/L at $t = 300$ minutes. This value of chlorine lies between minimum and maximum residual levels of 0.4 and 4.0mg/L for chloramine dosing, measured as Cl_2 and is below the maximum tolerable benchmark of 4 mg/L prescribed by Environmental Protection Agency(USEPA, 1999; 2006). It implies that values which are below maximum tolerable level are safe for human consumption and could be obtained from the mid-point of the last section of the system at $t = 300$ minutes.

5 Conclusion

In this paper, we modelled chlorine concentration decay profile through multiple-compartment isothermal reactor system. The system consists of both bulk fluid of water and Pseudomonas aeruginosa attached to the wall sieves in the reactor system. Figures 3 and 5 indicate penetration of chlorine through biofilm of P.aeruginosa for 18 minutes while Figures 8 and 10 represent penetration of chlorine through biofilm of P.aeruginosa with humic acid for 300 minutes.

Also transport of chlorine through bulk water zones are shown in Figures 2, 4, 6, 7, 9 and 11 at different times. From the results, constructed model effectively predicted chlorine concentration decay profile through bulk water and intermediate sections consisting of biofilm of P.aeruginosa with humic acid for system operated under isothermal condition.

References

Armstrong, A.M., Sobsey, M.D., Cassanova, Y. (2016). Disinfection of Escherichia Coli and Pseudomonas aeruginosa by copper in water. *Journal of Water and Health* 14, 424-432.

Biswas, P., Lu, C.S., Clark, R.M. (1993). Chlorine concentration decay in pipes. *Water Res.*, *27*(12), 1715 - 1724.

Chen, X., Stewart, P.S. (1996). Chlorine penetration into artificial biofilm is limited by a reaction- diffusion interaction. *Environmental science & technology*, *30*(6), 2078-2083.

Chick, H. (1908). An investigation of the laws of disinfection. *Journal of Hygiene*, 8, 92-158.

De Beer, D., Srinivasan, R., and Stewart, P.S. (1994). Direct measurement of chlorine penetration into biofilms during disinfection. *Applied and Environmental Microbiology,* 60, 4339-4344.

Kiene, L., Lu, W., Levi, Y. (1993). Parameters governing the rate of chlorine decay throughout distribution system. In *Proc. AWWA Annual Conference*. San Antonio, TX.

Levy, K., Nelson, K.L., Hubbard, A., Eisenberg, J.N. (2008). Following the water : a controlled study of drinking water storage in northern coaster Ecuador. *Environmental Health perspective*, *116*(1), 73 -80.

Lee, W.H., Pressman, J.G., and Wahman, D.G. (2018). Three-dimensional free chlorine and monochloramine biofilm penetration: Correlating penetration with biofilm activity and viability. *Journal of Environmental Science and Technology*, 52, 1889-1898.

Liou, C.P., Kroon, J.R. (1987). Modelling the propagation of waterborne substances in distribution networks. *Journal of American Water Works Association*, 79, 54-58.

Mao, G., Song, Y., Bartlam, M., Wang, Y. (2018). Long-term effects of residual chlorine on Pseudomonas aeruginosa in simulated drinking water fed with low AOC medium. *Journal of Frontiers of Microbiology* 9, 879. doi: 10.3389/fmicb.2018.00879.

Mau, R.E., Boulos, P.F., Bowcock, R.W. (1996). Modeling distribution storage water quality: An analytical approach. *Journal of Applied Mathematical Modelling*, 20, 329-338.

Mau, R.E., Boulos, P.F., Bowcock, R.W. (1996). Modeling distribution storage water quality: An analytical approach. *Journal of Applied Mathematical Modelling*, 20, 329-338.

Perry, R. H., and Chilton, C. H (1973). *Chemical engineers' handbook.* 5th ed. McGraw-Hill, New York.

Powell, J.C., West, J.R., Hallam, B. N., Forster, C.F., Simms, J.(2000). Performance of Various Kinetic Models for Chlorine Decay. *Journal of Water Resources Planning and Management, ASCE,* 126(1), 13-20.

Pabst, B., Pitts, B., Lauchnor, E., Stewart, P.S. (2016). Gel-entrapped staphylococcus aureus bacteria as models of biofilm infection exhibit growth in dense aggregates, oxygen limitation, antibiotic tolerance and heterogeneous gene expression. *Journal of Antimicrob Agents Chemother*, 60, 6294-6301. doi: 10.1128/AAC.01336-16.

Selleck, R.E., Saunier, B.M., Collins, H.F., (1978). Kinetics of bacterial deactivation with chlorine. *J. Environ. Eng. Div., ASCE*, 104, 1197-1212.

Sharp W.W., Pfeiffer J., Morgan M. (1991). *In situ chlorine decay rate testing. Proc. Water Quality Modeling in Distribution Systems.* AWWA Research Foundation, 3-5 Feb. Cincinnati, OH. pp. 311-322.

USEPA (1999). *Disinfection Profiling and Bench-marking Guidance Manual EPA 815-R-99-01.* United States Environmental Protection Agency, Office of Water (4607). http://www.epa.gov/safewater/mdbp/pdf/profile/benchpt1.pdf.

USEPA(2006). *National Primary Drinking Water Regulations.* Long Term 2 Enhanced Surface Water Treatment Rule, 71 FR Jan 5, 2006, vol. 71, No. 3.

Vasconcelos, J.J., Boulos, P.F., Grayman, W.M., Kiene, L., Wable, O., Biswas, P., Bhari, A., Rossman, L.A., Clark, R.M., Goodrich J.A. (1996). *Characterization and modeling of chlorine decay in distribution systems.* AWWA Research Foundation, ISBN: 0-89867-870-6.

Watson, H.E. (1908). A note on the variation of rate of disinfection with the change in concentration of the disinfectant. *Journal of Hygiene*, 8, 536-542.

Xu, J., Huang, C., Shi, X., Dong, S., Yuan, B., Nguyen, T.H. (2018). Role of drinking water biofilms on residual decay and trihalomethane formation: An experimental and modeling study. *Journal of Science of Total Environment*, 642, 516-525.